Automatic Control (1)

By

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Lecture (2)

Course Title: **Automatic Control (1)**

Course Code: **ELE 314**

Contact Hours: **4.**

= [2 Lect. + 2 Tut + 0 Lab]

Assessment:

Final Exam: 60%.

Midterm: 20%.

Year Work & Quizzes: 20%.

Experimental/Oral: 20%.

Textbook:

1- Benjamin C. Kuo " Automatic control systems" 9th ed., John Wiley & Sons, Inc. 2010.

2- Katsuhiko Ogata, "Modern Control Engineering", 4th Edition, 2001.

Course Description

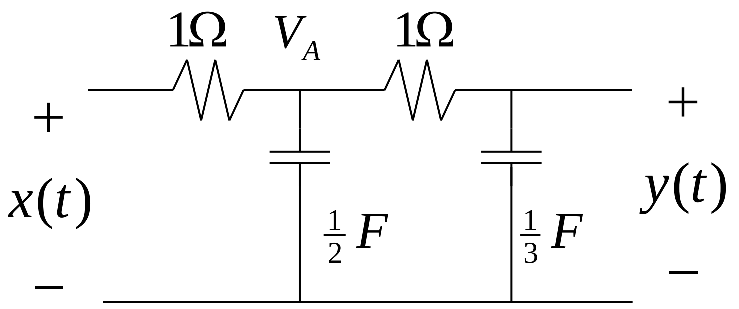
- State space variables, Solving state space equation, Basic definitions in modern control (Observability and Controllability), Transfer function analysis, Error analysis, Static and dynamic error coefficients, Steady state error, Error characteristics, Basic control action and industrial automatic control (P, PI, PID controllers), Transient response for control systems of first and second order, Poles / Zeros, Eigen value and stability of multivariable system, Stability analysis, Routh-Hurwitz criterion.

Stability

System Representations

- Continuous-time LTI system
  - Ordinary differential equation
  - Transfer function (Laplace transform)
  - Dynamic equation (Simultaneous first-order ODE)
- Discrete-time LTI system
  - Ordinary difference equation
  - Transfer function (Z-transform)
  - Dynamic equation (Simultaneous first-order ordinary difference equation)

Continuous-time LTI system



$$\frac{V_A - x(t)}{1} + \frac{1}{2}\frac{dV_A}{dt} + \frac{V_A - y(t)}{1} = 0$$

$$\Rightarrow 2V_A + \frac{1}{2}\frac{dV_A}{dt} = x(t) + y(t)$$

$$\frac{y(t) - V_A}{1} + \frac{1}{3}\frac{dy(t)}{dt} = 0$$

$$\Rightarrow y(t) + \frac{1}{3}\frac{dy(t)}{dt} = V_A$$

$$2[y(t) + \tfrac{1}{3}y'(t)] + \frac{1}{2}[y'(t) + \tfrac{1}{3}y''(t)] = x(t) + y(t)$$

$$\Rightarrow \tfrac{1}{6}y''(t) + \tfrac{7}{6}y'(t) + y(t) = x(t)$$

$$\Rightarrow y''(t) + 7y'(t) + 6y(t) = 6x(t)$$

Ordinary differential equation

Laplace transform

$$s^2Y(s) + 7sY(s) + 6Y(s) = 6X(s)$$

$$H(s) \equiv \frac{Y(s)}{X(s)}|I.C. = 0 = \frac{6}{s^2 + 7s + 6}$$

Transfer function

$$y''(t) + 7y'(t) + 6y(t) = 6x(t)$$

$$let \quad \begin{aligned} x_1(t) &= y(t) \\ x_2(t) &= y'(t) \end{aligned}$$

$$\Rightarrow x_2'(t) + 7x_2(t) + 6x_1(t) = 6x(t)$$

$$\Rightarrow x_1'(t) = x_2(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 6 \end{bmatrix} x(t)$$

State equation (Simultaneous first-order ODE)

Dynamic equation

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

output equation

Stability

- Internal behavior

  The effect of all characteristic roots.

- External behavior

  The effect by cancellation of some transfer function poles.

The Concept of Stability

A stable system is a dynamic system with a bounded response to a bounded input.

- Absolute stability is a stable/not stable characterization for a closed-loop feedback system.
- Given that a system is stable we can further characterize the degree of stability, or the relative stability.

Definition :

A system is internal (asymptotic) stable, if the zero-input response decays to zero, as time approaches infinity, for all possible initial conditions.

Asymptotic stable =>All the characteristic polynomial roots are located in the LHP (left-half-plane)

Definition :

A system is external (bounded-input, bounded-output) stable, if the zero-state response is bounded, as time approaches infinity, for all bounded inputs.

bounded-input, bounded-output stable =>All the poles of transfer function are located in the LHP (left-half-plane)

Asymptotic stable => BIBO stable
BIBO stable $\not\Rightarrow$ Asymptotic stable

System response

(i) First order system response

(ii)Second order system response

(iii)High order system response

The Routh-Hurwitz Stability Criterion

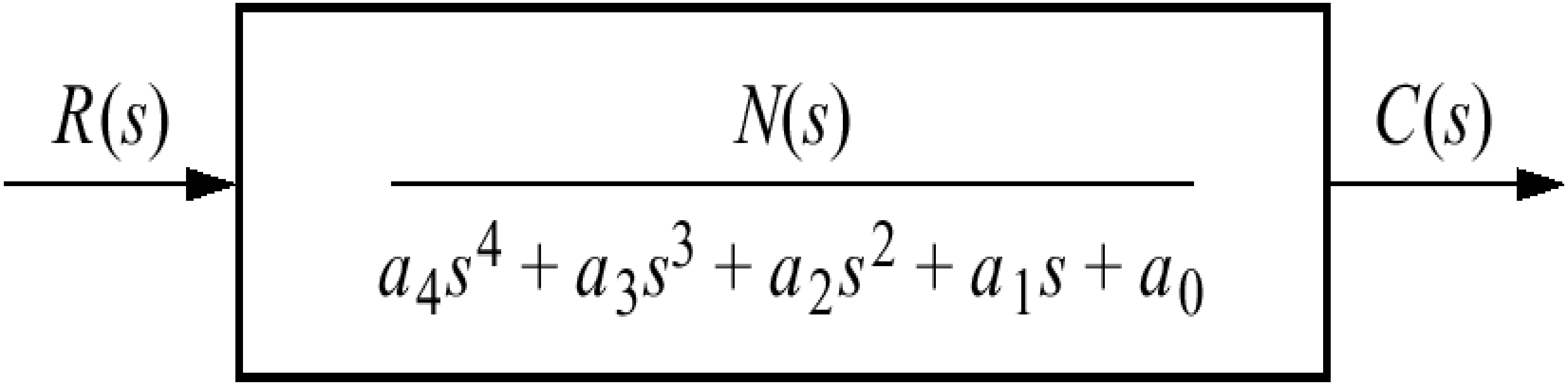
- It was discovered that all coefficients of the characteristic polynomial must have the same sign and non-zero if all the roots are in the left-hand plane.
- These requirements are necessary but not sufficient. If the above requirements are not met, it is known that the system is unstable. But, if the requirements are met, we still must investigate the system further to determine the stability of the system.
- The Routh-Hurwitz criterion is a necessary and sufficient criterion for the stability of linear systems.

The Routh-Hurwitz Stability Criterion Steps

The method requires two steps:

(1) Generate the data table (Routh table).

(2) Interpret the table to determine the number of poles in LHP and RHP.

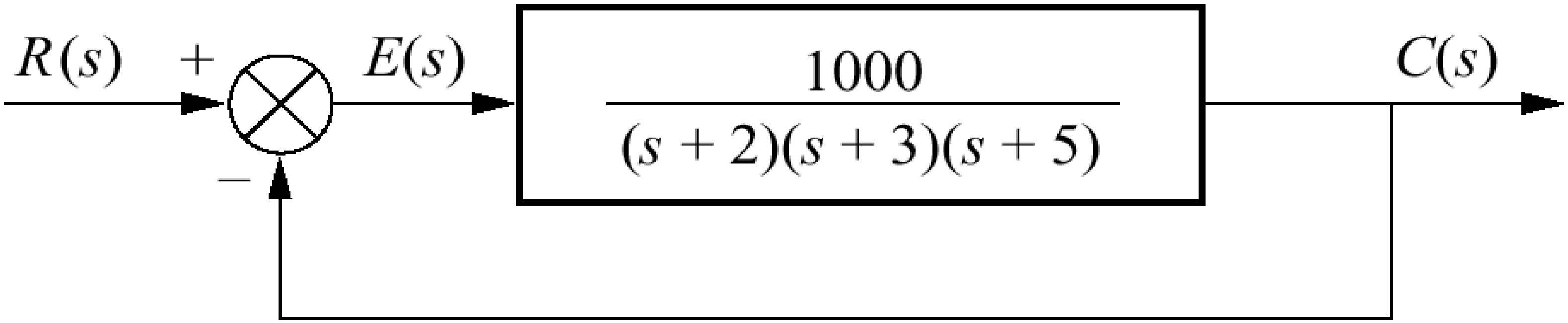


s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

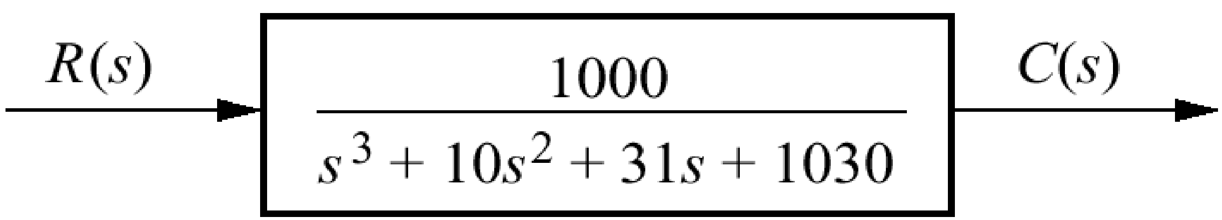
Completed Routh table

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Feedback system and its equivalent closed-loop system



(*a*)



(*b*)

Completed Routh table

s^3	1	31	0
s^2	$\cancel{10}\quad 1$	$\cancel{1030}\quad 103$	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$
s^0	$\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$

Interpretation of Routh table

The number of roots of the polynomial that are in the right half-plane is equal to the number of sign changes in the first column.

R-H: Special case. Zero in the first column

Replace the zero with ε , the value of ε is then allowed to approach zero from –ive or +ive side.

Problem Determine the stability of the closed-loop transfer function

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}\ \epsilon$	$\frac{7}{2}$	0
s^2	$\frac{6\epsilon - 7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	0
s^0	3	0	0

Label	**First Column**	$\epsilon = +$	$\epsilon = -$
s^5	1	$+$	$+$
s^4	2	$+$	$+$
s^3	$\cancel{0}\ \epsilon$	$+$	$-$
s^2	$\frac{6\epsilon - 7}{\epsilon}$	$-$	$+$
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	$+$	$+$
s^0	3	$+$	$+$

R-H: Special case. Entire row is zero

Problem Determine the number of right-half-plane poles in the closed transfer function $T(s) = \dfrac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$

Solution: Form an auxiliary polynomial, P(s) using the entries of row above row of zeros as coefficient, then differentiate with respect to s finally use coefficients to replace the rows of zeros and continue the RH procedure.

$$P(s) = s^4 + 6s^2 + 8 \qquad \frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

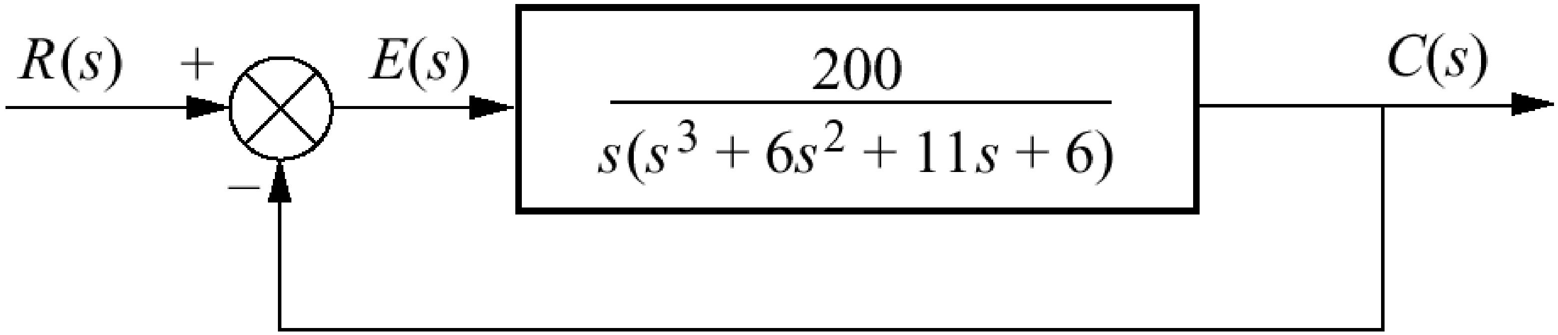
s^5	1	6	8
s^4	$\cancel{7}$ 1	$\cancel{42}$ 6	$\cancel{56}$ 8
s^3	$\cancel{0}$ $\cancel{4}$ 1	$\cancel{0}$ $\cancel{12}$ 3	$\cancel{0}$ $\cancel{0}$ 0
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
s^0	8	0	0

Problem Determine the number of poles in the right-half-plane, left-half-plan and on the $j\omega$ axis for the closed transfer function

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$-\cancel{10}\ \ -1$	$-\cancel{20}\ \ -2$	$\cancel{10}\ \ 1$	$\cancel{20}\ \ 2$	0
s^5	$\cancel{20}\ \ 1$	$\cancel{60}\ \ 3$	$\cancel{40}\ \ 2$	0	0
s^4	1	3	2	0	0
s^3	$\cancel{0}\ \ \cancel{4}\ \ 2$	$\cancel{0}\ \ \cancel{6}\ \ 3$	$\cancel{0}\ \ \cancel{0}\ \ 0$	0	0
s^2	$\cancel{\frac{3}{2}}\ \ 3$	$\cancel{2}\ \ 4$	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

Problem Determine the number of poles in the right-half-plane, left-half-plan and on the $j\omega$ axis for system



Solution: The closed loop transfer function is

$$T(s) = \frac{200}{s^4 + 6s^3 + 11s^2 + 6s + 200}$$

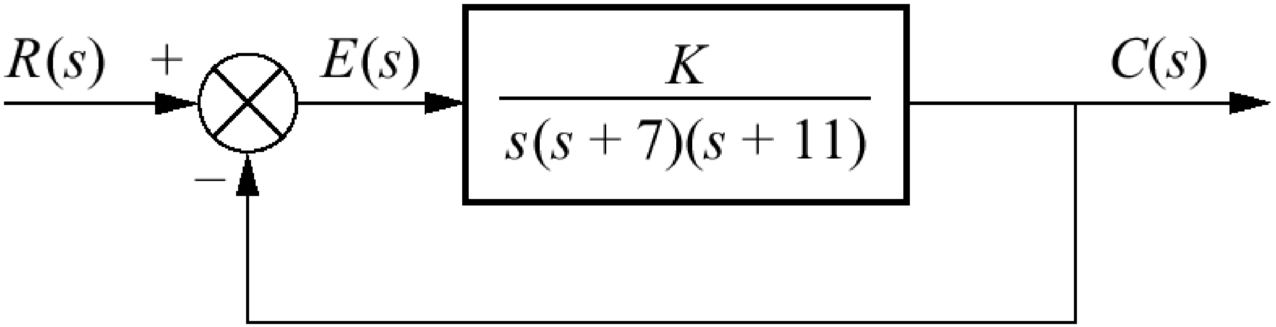
Routh table

s^4	1	11	200
s^3	$\cancel{6}$ 1	$\cancel{6}$ 1	
s^2	$\cancel{10}$ 1	$\cancel{200}$ 20	
s^1	-19		
s^0	20		

2 poles in RHP, 2 poles in LHP no poles on $j\omega$ axis
The system is unstable

Feedback control system

Problem Find the range of gain K for the system that will cuase the system to be stable, unstabel, and marginally stable. Assume K>0.



Solution: The closed loop transfer function is

$$T(s) = \frac{K}{s^3 + 18s^2 + 77s + K}$$

Routh table for Example 6.9

s^3	1	77
s^2	18	K
s^1	$\frac{1386-K}{18}$	
s^0	K	

s^3	1	77
s^2	18	1386
s^1	$\cancel{0}$ 36	
s^0	1386	

For K < 1386 the system is stable. For K > 1386 the system is unstable. For K = 1386 we will have entire row of zeros (s row). We form the even polynomial and differentiate and continue, no sign changes from the even polynomial so the 2 roots are on the $j\omega$ axis and the system is marginally stable

Routh table for Example 6.10

Problem Factor the polynomial $s^4 + 3s^3 + 30s^2 + 30s + 200$

Solution: from the Routh table we see that the s^1 row is a row of zeros. So the even polynomial at the s^2 row is $P(s) = s^2 + 10$ **since this polynomial is a factor of the original, dividing yields** $P(s) = s^2 + 3s + 20$ **As the other factor so**

$$s^4 + 3s^3 + 30s^2 + 30s + 200 = (s^2 + 10)(s^2 + 3s + 20)$$

s^4	1	30	200
s^3	$\cancel{3}$ 1	$\cancel{30}$ 10	
s^2	$\cancel{20}$ 1	$\cancel{200}$ 10	
s^1	$\cancel{0}$ 2	$\cancel{0}$ 0	
s^0	10		

Stability is State Space Example 6.11

Problem Given the system

$$\dot{X} = \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix} X + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X$$

Find out how many poles in the LHP, RHP and on the $j\omega$ **axis**

Solution: First form (sI-A)

$$(sI - A) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 8 & 1 \\ -10 & -5 & -2 \end{bmatrix} = \begin{bmatrix} s & -3 & -1 \\ -2 & s-8 & -1 \\ 10 & 5 & s+2 \end{bmatrix}$$

Now find the det (sI-A) = $s^3 - 6s^2 - 7s - 52$

Routh table for Example 6.11

s^3	1	-7
s^2	$-\cancel{6}\ -3$	$-\cancel{52}\ -26$
s^1	$-\cancel{\frac{47}{3}}\ -1$	$\cancel{0}\ \ 0$
s^0	-26	

One sign change, so 1 pole in the LHP and the system is unstable

With Our Best Wishes
Automatic Control (1)
Course Staff